Hw 4 - Solutions

① **Prove:** if $\sum_{n=1}^{\infty} a_n$ is convergent and $\sum_{n=1}^{\infty} b_n$ is divergent, then $\sum_{n=1}^{\infty} (a_n + b_n)$ is divergent.

Proof: let $\sum_{n=1}^{\infty} a_n \longrightarrow L$ and let $\sum_{n=1}^{\infty} b_n$ be divergent.

Assume by contradiction, that $\sum_{n=1}^{\infty} (a_n + b_n) \longrightarrow L'$.

then $$\sum_{n=1}^{\infty} (a_n + b_n) - \sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (a_n + b_n - a_n) = \sum_{n=1}^{\infty} b_n$$

and $$\sum_{n=1}^{\infty} (a_n + b_n) - \sum_{n=1}^{\infty} a_n = L' - L$$

thus $\sum_{n=1}^{\infty} b_n = L' - L$ so b_n is convergent $\Rightarrow\Leftarrow$ $\blacksquare$

② a) $\lim_{n\to\infty} \frac{n^2+2}{(n+1)^2} = 1 \neq 0$

thus by the Divergence Test, $\sum_{n=1}^{\infty} \frac{n^2+2}{(n+1)^2}$ diverges.

b) this series can be written $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} = \sum_{n=1}^{\infty} n^{-3/2}$

which is a p-series with $p = 3/2 > 1$ so the series converges

③ Consider $\sum_{n=1}^{\infty} \frac{1}{n}$